(Solns)

Math 2E Quiz 5 Morning - April 28th

Please write your name and ID on the front.

Show all of your work, and simplify all your answers. *There is a question on the back side.

1. Sketch the gradient vector field, ∇f, of the following function $f(x,y) = (x-y)^2$ by:

(a) First, find ∇f. When is $\nabla f = (0,0)$?

2pts

$\nabla f(x,y) = \langle 2(x-y), -2(x-y) \rangle$, so $\nabla f = 0$ when $x=y$.

+1 +1

(b) Since $f \geq 0$, its level surfaces are always non-negative. Solve the xy-level-set-curves from the level sets $f(x,y) \equiv k$, in other words from $(x-y)^2 = k$, for $k = 0, 1, 4, 9$. Note the LHS is squared!

3pts

$(x-y)^2 = k \implies x - y = \pm\sqrt{k}$ so $\underline{y = x \mp \sqrt{k}}$

With $k=0$:	$y = x$	
$k=1$:	$y = x \mp 1$	+1
$k=4$:	$y = x \mp 2$	+1
$k=9$:	$y = x \mp 3$	+1

All lines.

Only (−1) if forgot about $-\sqrt{k}$.

3pts

(c) Draw the level set curves with $k = 0, 1, 4, 9$ from (b) on the given plot. Label them, too.

(d) On the same plot, also sketch the gradient vector field ∇f based on your answers from (b) and (c). Use a small "o" along wherever the gradient is zero.

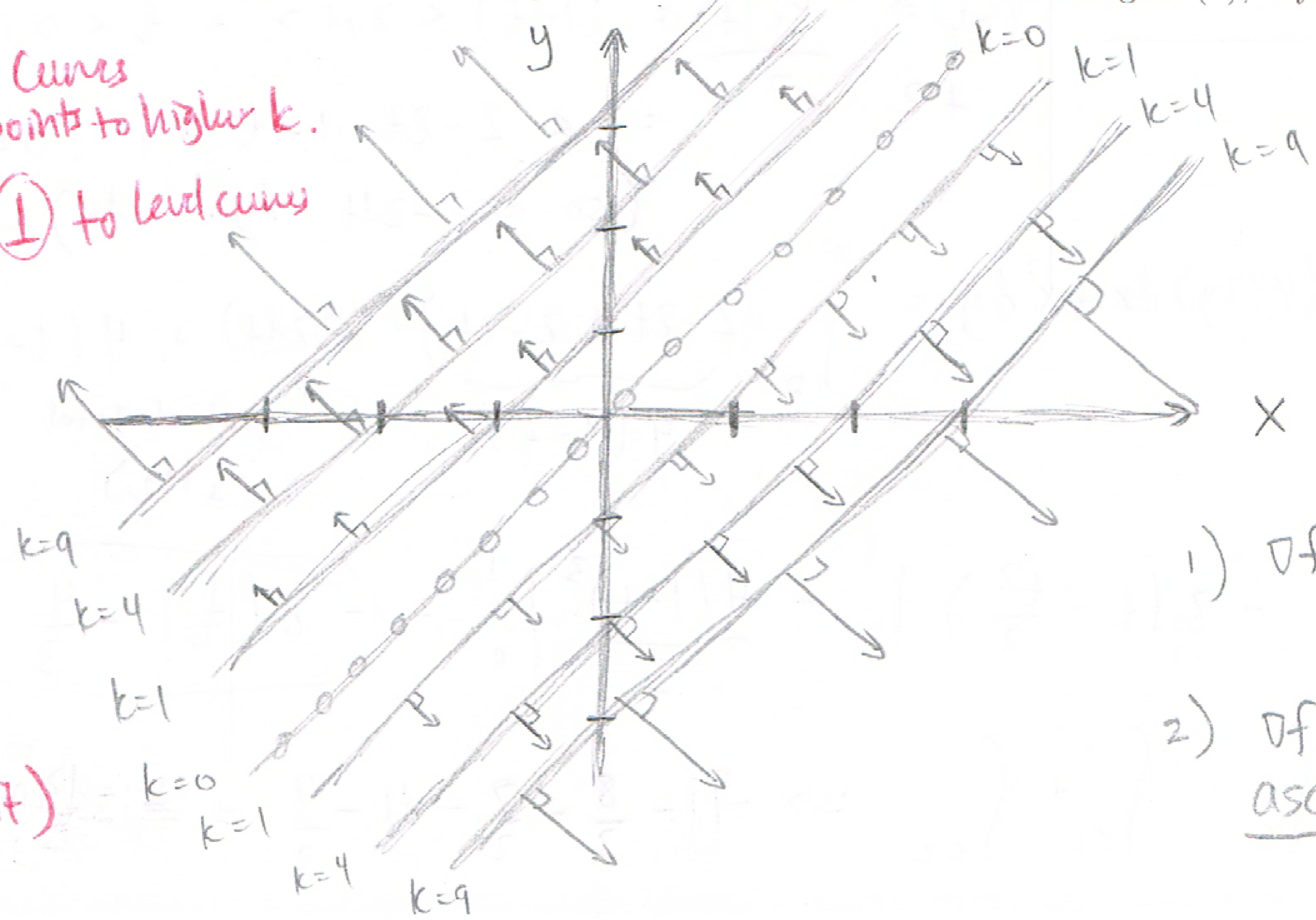
As a means of checking your work, does your result look sensible from looking at (a), ∇f?

+1 : Level Curves

+1 : ∇f points to higher k.

+1 : ∇f $\perp$ to level curves

Bonus Pt: If ∇f is getting bigger is drawn. (only for #1 if missing credit)



1) ∇f $\perp$ to level sets

2) ∇f points in ascending k-value

Note: For #3, you could redo your HW problem since arclength integrals are independent of C's parameterization.

3 pts

2. Set up the integral that would evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = < x, y, xy >$ and the curve C is parameterized by $\mathbf{r}(t) = < \cos(t), \sin(t), t >$ with $0 \leq t \leq \pi$. Don't evaluate it!

- $\vec{r}'(t) = < -\sin t, \cos t, 1 >$ +1
- $\vec{F}(\vec{r}(t)) = < \cos t, \sin t, \cos t \sin t >$ +1

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{\pi} \left(\cos t(-\sin t) + \sin t \cos t + \cos t \sin t\right) dt = \int_0^{\pi} \cos t \sin t \, dt.$$

+1 either.

9 pts

3. Compute $\int_C xe^{yz} ds$ where C is the line segment from (1,2,3) to (0,0,0).

(i) $C \Rightarrow \vec{r}(t) = (1-t) < 1,2,3 > + t < 0,0,0 >$

+2 $= < 1-t, 2(1-t), 3(1-t) >, \quad (0 \leq t \leq 1).$

Thus, $x'(t) = -1, \quad y'(t) = -2, \quad z'(t) = -3$ +2

(ii) $\displaystyle\int_C xe^{yz} ds = \underbrace{\int_0^1 (1-t)\, e^{6(1-t)^2}}_{+2} \cdot \sqrt{1+4+9}\, dt$

$\boxed{u = (1-t)^2 \quad du = -2(1-t)\,dt}$

(total of 4 pts for right integral)

$$= \sqrt{14} \int_{u=1}^{u=0} e^{6u} \cdot \frac{du}{-2} = -\frac{\sqrt{14}}{2} \cdot \frac{e^{6u}}{6} \Big|_{u=1}^{u=0}$$

$$= -\frac{\sqrt{14}}{12}\left(e^0 - e^6\right) = \boxed{\frac{\sqrt{14}}{12}\left(e^6 - 1\right)} \quad +3$$

(2 pts for right u-sub, 1 pt computing)

Note: Even though we traveled backwards along C, the value is the same (as on HW) ✓